# Accounting for Group Classification Error in Variance Estimates Using the American Community Survey 

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## Motivation

- Wanted to classify occupations as "high" or "low" wage jobs.
- Great! ACS can do that!
- Calculate median earnings for individual occupations. Set a criteria. Define occupations.
- But how certain am I about that classification?
- How do I account for that uncertainty in my final estimate?


## Outline

- General case
- Naïve method
- Sophisticated method
- Example using areas of concentrated poverty
- Simulation


## General Case

- Individuals are arranged in $K$ groups
- For each group $K=k$, a statistic $\theta_{k}$ is calculated with standard error $\sigma_{k}$.
- $\theta_{k}$ is compared against some critical value $\tau$ and an indicator $y_{k}$ is set to 1 or 0 .

$$
y_{k} \begin{cases}=0, & \text { if } \theta_{k} \leq \tau \\ =1, & \text { if } \theta_{k}>\tau\end{cases}
$$

## General Case Naïve Variance

- Final estimate is $Y=\sum\left(y_{k} W_{i}\right) / \sum W_{i}$
- Naïve variance

$$
\begin{aligned}
\tilde{Y}_{r} & =\sum\left(y_{k} W_{i, r}\right) / \sum W_{i, r} \\
\sigma_{\tilde{y}}^{2} & =\widetilde{\operatorname{var}}(Y)=\frac{\sum\left(\tilde{Y}_{0}-\tilde{Y}_{r}\right)^{2}}{R(1-\varepsilon)^{2}}
\end{aligned}
$$

## General Case Measurement Error

- The assignment of $y_{k}$ is a measurement error problem

$$
\begin{gathered}
y=\hat{y}+\eta, \quad \eta \sim N\left(0, \sigma_{\eta}\right) \\
\sigma_{y}^{2}=\sigma_{\hat{y}}^{2}+\sigma_{\eta}^{2}+2 \sigma_{\hat{y}, \eta}
\end{gathered}
$$

- $\sigma_{\eta}^{2}$ is related to the variances of $\theta_{k}$


## General Case Sophisticated Variance

$$
\begin{gathered}
y_{k, r} \begin{cases}=0, & \text { if } \theta_{k, r} \leq \tau \\
=1, & \text { if } \theta_{k, r}>\tau\end{cases} \\
Y_{r}=\sum\left(y_{k, r} W_{i, r}\right) / \sum W_{i, r} \\
\sigma_{y}^{2}=\operatorname{var}(Y)=\frac{\sum\left(Y_{0}-Y_{r}\right)^{2}}{R(1-\varepsilon)^{2}}
\end{gathered}
$$

## General Case

## Measurement Error (Cont.)

- Variance attributed to indicator alone

$$
\begin{gathered}
\delta_{k, r}=y_{k, 0}-y_{k, r} \\
\Delta_{r}=\frac{\sum\left(\delta_{k, r} W_{i, r}\right)}{\sum W_{i, r}}=\tilde{Y}_{r}-Y_{r} \\
\sigma_{\eta}^{2}=\operatorname{var}(\Delta)=\frac{\sum\left(\Delta_{0}-\Delta_{r}\right)^{2}}{R(1-\varepsilon)^{2}}
\end{gathered}
$$

## How they relate

$$
\begin{aligned}
\sigma_{y}^{2}=\operatorname{var} & (Y)=\frac{\sum_{R}\left(Y_{0}-Y_{r}\right)^{2}}{R(1-\varepsilon)^{2}}=\frac{\sum_{R}\left(Y_{0}-\widetilde{Y}_{r}+\widetilde{Y}_{r}-Y_{r}\right)^{2}}{R(1-\varepsilon)^{2}} \\
& =\frac{\sum_{R}\left[\left(Y_{0}-\widetilde{Y}_{r}\right)^{2}+\left(\widetilde{Y}_{r}-Y_{r}\right)^{2}+2\left(Y_{0}-\widetilde{Y}_{r}\right)\left(\widetilde{Y}_{r}-Y_{r}\right)\right]}{R(1-\varepsilon)^{2}} \\
& =\frac{\sum_{R}\left(Y_{0}-\widetilde{Y}_{r}\right)^{2}}{R(1-\varepsilon)^{2}}+\frac{\sum_{R}\left(\widetilde{Y}_{r}-Y_{r}\right)^{2}}{R(1-\varepsilon)^{2}}+2 \frac{\sum_{R}\left(Y_{0}-\widetilde{Y}_{r}\right)\left(\widetilde{Y}_{r}-Y_{r}\right)}{R(1-\varepsilon)^{2}} \\
& =\sigma_{\hat{y}}^{2}+\sigma_{\eta}^{2}+2 \sigma_{\hat{y}, \eta}
\end{aligned}
$$

## Applications

- Industries as generous providers of health insurance
- Foreign born groups (by country of birth) as "new/emerging" immigrants
- Neighborhoods as impoverished
- Bishaw, 2011
- Etc...


## Poverty Areas Example

- Areas With Concentrated Poverty: 2006-2010
- ACS Brief that examines census tracts by poverty rate:
Category I (0-13.7\%)
Category II (13.8\%-19.9\%)
Category III (20.0\%-39.9\%)
Category IV (40.0\%-100.0\%)


## Methods

- 72,254 Census tracts in U.S.
- Used 2006-2010 ACS 5-year data to calculate poverty rates for tracts
- Standard errors calculated using replicate weights.
- 517 tracts had rates of 0 percent and 18 had rates of 100 percent
- Standard errors calculated using ACS Production method (based on tract size and average weight in the state)
- Replicate poverty rates simulated from SE

100 Random Tracts


## Poverty Rates

| Tract Group | Number <br> of Tracts | Percentage of <br> Population |
| :--- | ---: | ---: |
| Category I | 42,383 | 61.4 |
| Category II | 11,574 | 16.0 |
| Category III | 14,823 | 19.1 |
| Category IV | 3,474 | 3.5 |

## Tract CVs


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## Results

|  | Category <br> I | Category <br> II | Category <br> III | Category <br> IV |
| :---: | :---: | :---: | :---: | :---: |
| Naïve variance (ひ) | 0.000153 | 0.000079 | 0.000118 | 0.000025 |
| Naïve standard error | 0.012355 | 0.008887 | 0.010850 | 0.005017 |
| Sophisticated variance (var $(Y)$ ) | 0.058874 | 0.223126 | 0.023230 | 0.071863 |
| Sophisticated standard error | 0.242639 | 0.472362 | 0.152413 | 0.268073 |
| Measurement error variance $(\operatorname{var}(\Delta))$ | 0.062034 | 0.225113 | 0.024111 | 0.071109 |
| Covariance $(\operatorname{cov}(Y, \Delta))$ | -0.001656 | -0.001033 | -0.000499 | 0.000364 |
| Ratio of standard errors | 19.64 | 53.15 | 14.05 | 53.43 |

## Size of Standard errors


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## State Estimates

|  | Category I | Category II | Category III | Category IV |
| :--- | :---: | :---: | :---: | :---: |
| Median CV <br> (Naïv) | 0.001 | 0.004 | 0.004 | 0.014 |
| Median CV <br> (sophisticated) | 0.029 | 0.118 | 0.085 | 0.190 |
| Smallest Ratio of <br> Standard Errors | 12.2 | 18.7 | 12.1 | 7.1 |
| Largest Ratio | 37.5 | 55.0 | 34.6 | 30.2 |
| Median Ratio | 19.9 | 29.4 | 17.4 | 13.7 |

## Simulation

- Attaching to other datasets
- Different number of replicates
- Can't get replicate estimates from public use data
- Use FactFinder Estimates/Standard Errors
- Simulate the Replicate Distribution
- Normal distribution $\sim N\left(\theta_{k}, \gamma \sigma_{k}^{2}\right), \gamma=\frac{R(1-\varepsilon)^{2}}{(R-1)}$


## Simulated and Replicate Based Standard Errors - States


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## Conclusion

- Error can be quite large!!
- Provide greater utility to working with estimates for small domains as an aggregate
- Properly reflect the level of uncertainty associated with estimates

SAS code available in an appendix to the paper

## Thank You!

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